

Analyzing Keplerian Elements—Part 1

Most satellite operators just use Keplerian elements in automatic satellite tracking programs. But you can tell a lot about an orbit from Keplerian elements and some relatively simple mathematics.

Most amateur satellite operators use Keplerian elements for pass predictions and in real-time satellite tracking software. But to some of us, orbital parameters are mysterious mathematical things that take a “rocket scientist” to understand. You might be pleasantly surprised to find out that the Keplerian elements tell you some very useful things about a satellite’s orbit, including how high the satellite orbits, what the longest distance is that you can

communicate via the satellite, and whether or not you can expect communications opportunities with the satellite at any given time.

All of these things can be computed with some relatively simple equations I’ve included in this article. WAIT—don’t stop reading! These equations are simple enough for a basic scientific calculator, and I also posted an Excel® spreadsheet on my Web page (<<http://www.mindspring.com/~n2wwd>>; see

Figure 1) that you may use to do the same computations that I do in this article and in next month’s Part 2. All you need to do is input the satellite’s Keplerian elements and your own latitude, and the spreadsheet will do the rest. The next section of this article will show you how to read the Keplerian elements so you can either do the calculations or put them into the spreadsheet.

What Are Keplerian Elements?

We first discussed Keplerian elements in the August, 1998, “Orbital Elements” column. This discussion included the illustrations in Figure 2 and Figure 3, which show the “physical” meaning of the Keplerian elements and how they relate to Kepler’s laws of orbital motion.

← Figure 1. Sample of the Excel® spreadsheet posted on N2WWD’s Web site (<<http://www.mindspring.com/~n2wwd>>), which you may use to calculate characteristics of a satellite’s orbit, based on its Keplerian elements. Again, the figures given are for AMSAT-OSCAR-10.

Keplerian Element Input		
Satellite:	AO-10	
Catalog Number:	14129	
Epoch Time:	99020.76950000	
Element Set	5	
Inclination:	26.9297	deg
RA of Node:	46.9808	deg
Eccentricity:	0.6011222	
Arg of Perigee:	284.2856	deg
Mean anomaly:	119.1380	deg
Mean motion:	2.05869872	rev/day
Decay rate:	0.00E+00	rev/day^2
Epoch Rev:	11735	
Checksum:	279	

Orbital Characteristics			
Orbital Period	699.5	minutes	
Semimajor Axis	26102	kilometers	
Apogee Altitude	35421	kilometers	
Perigee Altitude	4040	kilometers	
Ground Range	7128	kilometers (Maximum Communications Distance)	
Guaranteed Coverage	79.2	deg - Latitude (North or South)	
Maximum Coverage	90.0	deg - Latitude (North or South)	

Coverage	
42.4	(%) at Apogee
19.4	(%) at Perigee

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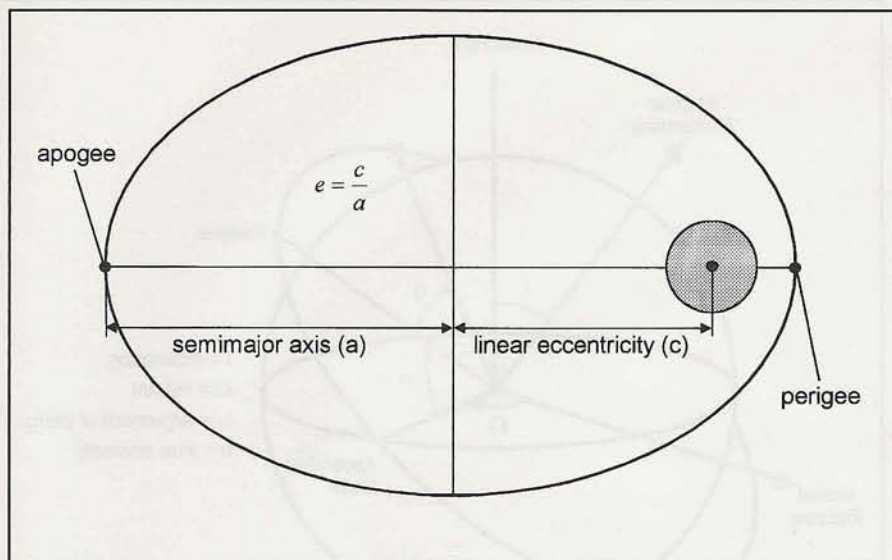


Figure 2. Kepler's First Law says satellites orbit in elliptical paths, with the Earth at one focus, or special off-center point, of the ellipse. The first two Keplerian elements, the semimajor axis and the linear eccentricity, describe the size and shape of the ellipse.

That column also included a discussion of the common NORAD/NASA Two-Line Element (TLE) format, and the more verbose AMSAT Keplerian element format, in which the elements are distributed. Since that was a year ago, we'll briefly recap here before digging in.

Kepler's First Law states that satellites orbit in elliptical paths with the Earth at a special off center point along the ellipse's longest dimension (known as a focus). Figure 2 illustrates the semimajor axis (a) as half the longest dimension of the ellipse. The eccentricity (e) is the shape of the orbit, described as the distance of the Earth from the center of the ellipse (linear eccentricity) divided by the semimajor axis. A circular orbit would have a zero eccentricity; the eccentricity of a very elongated orbit can almost reach one.

Figure 3 shows that the remaining four elements are angles that orient the orbital

ellipse relative to the Earth. Two of the angles orient the plane of the orbital ellipse. The inclination (i) is the angle between the ellipse's plane and the equator. Consequently, the inclination also tells you the northernmost and southernmost latitudes over which the satellite will directly fly. The other planar angle is the right ascension of the ascending node (Ω), or RAAN. This is the angle measured eastward from a fixed direction in space called the Vernal Equinox to the satellite's south-to-north equatorial crossing (called the ascending node).

The other two angles are orientations within the plane of the orbital ellipse. The argument of perigee (ω) is the angle from the ascending node to perigee in the direction of satellite motion. The true anomaly (θ) is the angle from perigee to the satellite's position at the epoch time listed in the Keplerian elements. Keplerian elements actually use a variant of true

"A satellite's Earth surface coverage increases as the altitude increases; high-altitude satellites approach 50% Earth coverage....In general, the more surface coverage a satellite has, the more communications and long distance (DX) opportunities there are."

anomaly called mean anomaly (with "mean" meaning "average" in this context). Using mean anomaly makes it a little bit easier to compute the satellite's position for a desired time. Epoch time, by the way, refers to standard calendar date and time.

As noted above, Keplerian elements are generally distributed in either or both of two formats, TLE (see Table 1) and AMSAT (see Table 2). Both the TLE and AMSAT Keplerian parameters are known as mean elements; they are similar, but not exactly identical, to the physical parameters known as osculating elements, shown in Figures 2 and 3. This distinction, while relatively unimportant to the computations performed in this article, is very important when choosing an orbit computation technique (i.e., orbit propagator) in a satellite-tracking program.

High and Low: Apogee and Perigee

Given an osculating mean motion, you can compute the orbit's semimajor axis and orbital period (the time it takes a satellite to make one complete revolution around the Earth). From the semimajor axis and eccentricity, you can in turn compute the orbit's highest altitude, or apogee, and its lowest altitude, or

Table 1. Two-Line Element (TLE) Format

AO-10									
1	14129U	83058B	99020.76950000	.00000000	00000-0	55553-5	0	54	
2	14129	26.9297	46.9808	6011222	284.2856	119.1380	2.05869872117355		

Table 1. The TLE, or Two-Line Element, format in which NASA releases Keplerian elements for various satellites. The one shown in this example is AMSAT-OSCAR-10. This format is ideal for entering into computer tracking programs, since you don't need to know what all the numbers mean (the program does).



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Oops...

In last month's Part 2 of KF6FGB's "Portable Power" series, we forgot to include one supplier of high-capacity 12-volt gel-cells designed for ham use (so you won't have to figure out what to do with the jumper cables on the ones designed for cars!). So, in addition to your local auto parts store, you might want to check out The Ham Contact, P.O. Box 4025, Westminster, CA 92684; Phone: (800) 933-4264; Fax: (714) 901-0583; E-mail: n6yyo@sprynet.com.



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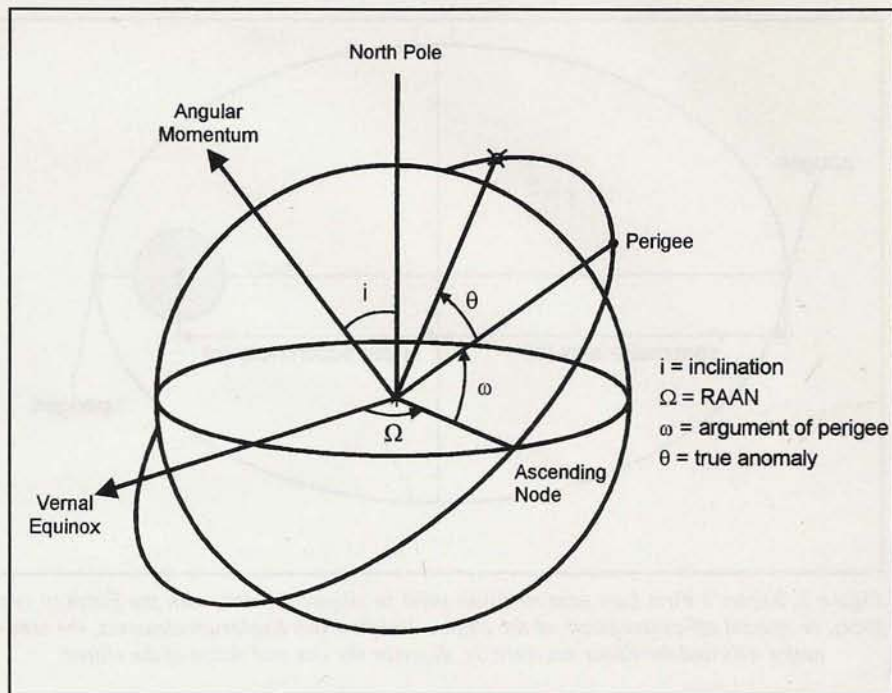


Figure 3. The remaining four Keplerian elements (beyond the two shown in Figure 2) are angles that orient the orbital ellipse relative to the Earth.

Table 2. AMSAT Keplerian Element Format

```
Satellite: AO-10
Catalog number: 14129
Epoch time: 99020.76950000
Element set: 5
Inclination: 26.9297 deg
RA of node: 46.9808 deg
Eccentricity: 0.6011222
Arg of perigee: 284.2856 deg
Mean anomaly: 119.1380 deg
Mean motion: 2.05869872 rev/day
Decay rate: 0.00000e+00 rev/day^2
Epoch rev: 11735
Checksum: 279
```

Table 2. The longer AMSAT format for Keplerian elements offers an explanation of each set of numbers. By reading this column, you should now know what most of them mean! This is useful for both computer and manual satellite tracking.

perigee. In the previous section, I made it a point to distinguish the *mean* elements given in the TLE and AMSAT formats from the purely physical (*osculating*) quantities. The purpose was to let you know that the computations we're making here are not exact. The differences between mean and osculating elements are small enough that the computations we make here, while not accurate enough for precision tracking, are "close enough"

for the characterizing descriptions we're making about the orbit.

The mean motion is the orbital frequency in revolutions per day. Since there are 1,440 minutes per day (24 hours per day x 60 minutes per hour) the time (in minutes) to make one revolution (the period) is easily computed:

$$\text{Period} = \frac{1440}{\text{Mean\_Motion}}$$

We need to know the value of the semimajor axis in order to compute the apogee and perigee heights, using the following formula (*remember, folks—you can use a scientific calculator or the spreadsheet on Ken's Web page to actually do these calculations.—ed.*).

$$\text{Semimajor\_Axis} = \sqrt[3]{\frac{7.537122734 \times 10^{13}}{\text{Mean\_Motion}^2}}$$

The apogee and perigee altitudes (in kilometers) can be computed using the semimajor axis, the eccentricity, and subtracting the mean Earth radius, since we know the geometry of an ellipse:

$$\text{Apogee\_Altitude} = \text{Semimajor\_Axis} \cdot (1 + \text{Eccentricity}) - 6371$$

$$\text{Perigee\_Altitude} = \text{Semimajor\_Axis} \cdot (1 - \text{Eccentricity}) - 6371$$

A satellite's Earth surface coverage increases as the altitude increases; high-altitude satellites approach 50% Earth coverage. You can compute the percentage of Earth surface coverage using the satellite's altitude (in kilometers); and you can compute the coverage extremes using the apogee and perigee altitudes. The percentage of Earth surface coverage is computed by:

$$\text{Coverage\_}\% = 50 \cdot \left[\frac{\text{Altitude}}{\text{Altitude} + 6371} \right]$$

In general, the more surface coverage a satellite has, the more communications and long distance (DX) opportunities there are.

Latitude Coverage by Inclination

An orbit's *inclination* determines the most northern and southern latitudes over which the satellite will directly fly. By also considering the satellite's altitude at these extremes, we can determine the latitude limits in which stations will have some coverage for the satellite. This is most easily done for nearly circular orbits, since the altitude is more or less constant. It's more difficult for elliptical orbits because we would need to make some calculations using the orientation of the apogee-perigee (*apsidal*) line (i.e., argument of perigee) to determine the actual altitude at the most northerly and southerly latitudes (we'll defer these computations until Part 2).

For now, we'll compute the limiting cases, which are (a) latitudes *guaranteed* to have some coverage, using the perigee altitude, and (b) latitudes that could *never* have coverage, using the apogee altitude.

“An orbit's inclination determines the most northerly and southerly latitudes over which the satellite will directly fly. By also considering the satellite's altitude at these extremes, we can determine the latitude limits in which stations will have some coverage....”

The maximum latitude for satellite access is computed by the following three steps:

1) Determine the maximum overflight latitudes using the inclination. If the inclination is 90 degrees or less, that's the maximum overflight latitude. Otherwise, the maximum overflight latitude is 180 degrees minus the inclination.

2) Determine the footprint latitude range for the satellite's altitude (in degrees), using the following equation:

$$\text{Footprint\_latitude\_range} = \cos^{-1} \left(\frac{6371}{\text{Altitude} + 6371} \right)$$

3) Determine the most northerly and southerly possible latitudes for the given altitude by adding the footprint latitude range to the maximum overflight latitude. (Note that if the sum of the footprint latitude range and the maximum overflight latitude exceeds 90 degrees, the satellite has full global coverage at that altitude.)

The above procedure needs to be done only once for nearly circular orbits. For elliptical orbits, you should do the computation twice: once at perigee and once at the apogee altitude. Stations at any latitude below the result computed for perigee are guaranteed to have some pass opportunities for the satellite. Stations at any latitude above the result computed for apogee could never have any pass opportunities. Stations between the results computed for perigee and apogee might or might not have pass opportunities, depending on the orientation of the apsidal line.

“There's one other piece of useful information to obtain from these calculations: how far you can communicate. The longest distance you can communicate through the satellite can be derived from twice the footprint latitude range.”

There's one other piece of useful information to obtain from these calculations: how far you can communicate. The longest distance you can communicate through the satellite can be derived from twice the footprint latitude range. We already saw that the footprint ground range depends on the satellite's altitude, so the longest possible communications distance is computed from the footprint latitude range:

$$\text{Maximum\_communications\_range} = 2 \cdot \frac{\pi}{180} \cdot \text{Footprint\_latitude\_range}$$

In Summary

This article was, in part, a refresher on the Keplerian elements, emphasizing what they tell you about the satellite's orbit. We also learned to compute how long it takes the satellite to orbit the Earth (i.e., period) and how to compute the satellite's highest (apogee) and lowest (perigee) altitudes. We learned how to compute the longest possible line-of-sight communications distance (footprint diameter) and saw how this gives us bounds on the satellite's latitude coverage. These calculations will be expanded upon a little next month. ■



Analyzing Keplerian Elements—Part 2

In this follow-up to last month's introduction, N2WWD expands our exploration of Keplerian Elements to see how they're used to predict actual coverage range for satellites in elliptical orbits.

Last month, we saw how you could learn some key characteristics about an orbit by doing some relatively simple calculations with the Keplerian elements. This article extends these calculations, giving you a better idea of an elliptical orbit's actual ground coverage.

Introduction

The calculations we did in Part 1 determined the basic characteristics of an orbit from its Keplerian elements. The article included the equations needed to do the calculations, which are also coded in a Microsoft Excel® spreadsheet that you can download from the Keplerian Elements page of my Web site at <http://www.mindspring.com/~n2wwd>. The information in Part 1 was great for a circular orbit, but didn't tell the whole story for elliptical orbits.

There was uncertainty for elliptical orbits because we only set up latitudes ranging from "guaranteed" line-of-sight coverage to maximum line-of-sight coverage. The "guaranteed coverage" tells you that stations between the equator and the computed latitude will *definitely* have some line-of-sight pass opportunities. The "guaranteed coverage" latitude is pessimistic since it presumes the orbit's perigee (lowest altitude) is at the *least* favorable location on the orbit. Likewise, the "maximum coverage" in Part 1 is overly optimistic since it tells you the highest latitude that could *possibly* contact the satellite if the orbit's apogee (highest altitude) were in the *most* favorable location on the orbit.

No doubt you're really interested in the orbit's *actual* coverage rather than having a range of latitudes for which you can only say "maybe and maybe not" to line-

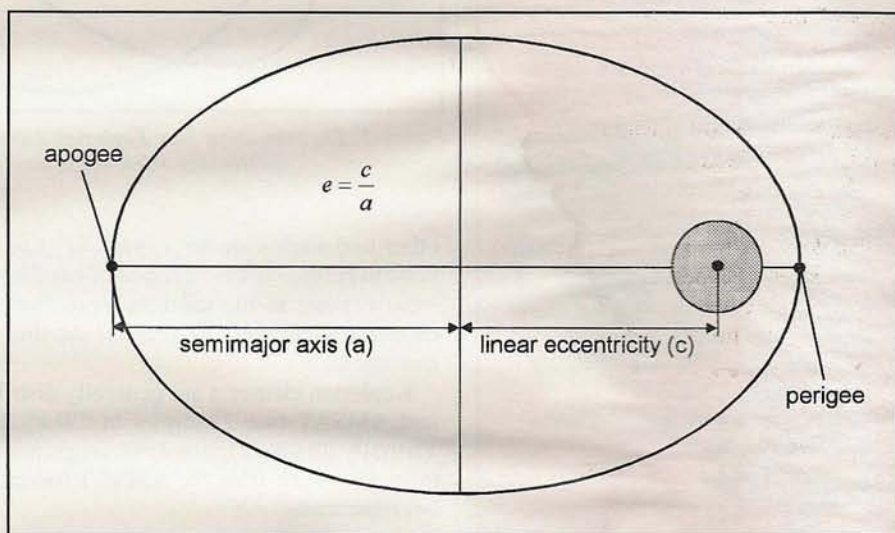


Figure 1. Kepler's First Law says satellites orbit in elliptical paths, with the Earth at one focus, or special off-center point, of the ellipse. The first two Keplerian elements—the semimajor axis and the linear eccentricity—describe the size and shape of the ellipse.

of-sight pass opportunities. This article, therefore, extends the calculations from Part 1 so you can determine the *actual* range of latitudes with pass opportunities. Naturally, the Excel spreadsheet on my Web site has been updated with the extended calculations.

What Are Keplerian Elements?

We discussed Keplerian elements in some detail last month and in the August, 1998, "Orbital Elements" column, so we'll just review them briefly this month. As we do, you might want to refer to Figures 1 and 2, which can give you a better visual understanding of the terms involved. Figure 1 shows two important parts of an orbit: the *semimajor axis* (a),

which is half the longest dimension of the ellipse, and the *eccentricity* (e), which is the shape of the orbit, described as the distance of the Earth from the center of the ellipse (linear eccentricity) divided by the semimajor axis. A circular orbit would have a zero eccentricity; the eccentricity of a very elongated orbit can almost reach one.

Figure 2 shows the remaining four elements—all angles that orient the orbital ellipse relative to the Earth, starting with the *inclination* (i), or angle between the ellipse's plane and the equator; and the *right ascension of the ascending node* (Ω), or RAAN, which is the angle measured eastward from a fixed direction in space called the *Vernal Equinox* to the satellite's south-to-north equatorial crossing (called the *ascending node*). The

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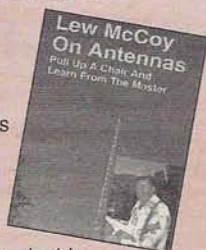
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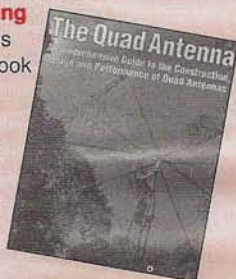
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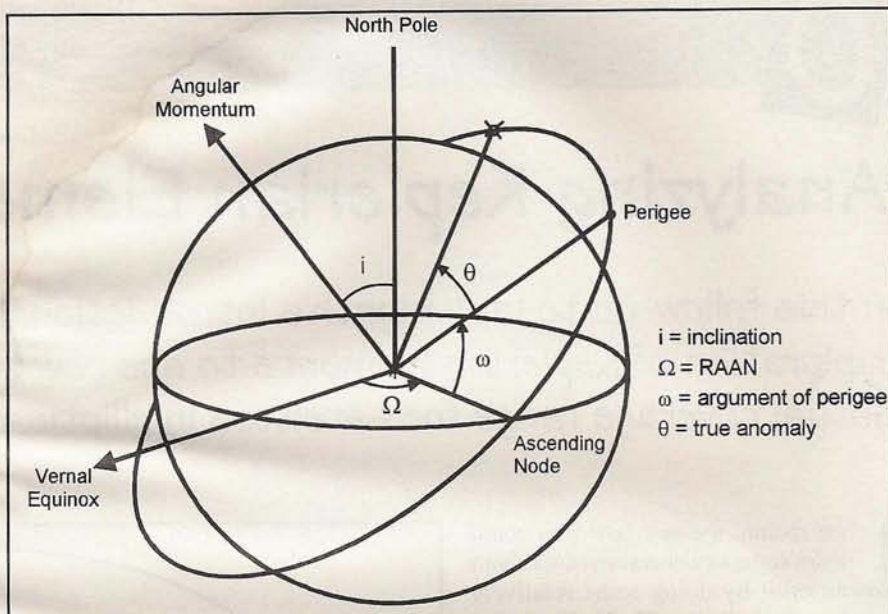


Figure 2. The remaining four Keplerian elements (beyond the two shown in Figure 3) are angles that orient the orbital ellipse relative to the Earth.

other two angles are the *argument of perigee* (ω), or the angle from the ascending node to perigee in the direction of satellite motion, and the *true anomaly* (θ), the angle from perigee to the satellite's position at the *epoch time* listed in the Keplerian elements (note that, for amateur satellite, *mean anomaly* figures are used instead of *true anomaly*).

Keplerian elements are generally distributed in either or both of two formats, TLE and AMSAT (see examples in "Worksheet for Analyzing Keplerian Elements"). Virtually all satellite tracking programs will use one or the other. Manual calculations are easier using the AMSAT format, which explains which element each number represents.

High and Low: Apogee and Perigee

Following up on last month's look at determining coverage for circular orbits and approximations of coverage for elliptical orbits, we'll now extend the calculations to get more exact in our examinations of elliptical orbits.

The first thing we need to do when extending the coverage calculations for elliptical orbits is to locate the apogee and perigee. The argument of perigee (ω) specifies the orbit's apogee-perigee line orientation. We compute the perigee latitude using the inclination and the argument of perigee as follows:

$$\text{perigee\_latitude} = \sin^{-1} [\sin(\text{inclination}) \cdot \sin(\text{argument\_of\_perigee})]$$

Very conveniently, the apogee latitude is just the negative of the perigee latitude. Note that positive latitude is in the Northern Hemisphere and negative latitude is in the Southern Hemisphere.

Now that we know the apogee latitude, we can determine the northernmost and southernmost latitudes that have a line of sight to apogee. The northernmost and southernmost apogee visibilities are computed by:

$$\text{northern\_apogee\_visibility} =$$

$$\text{apogee\_latitude} + \sin^{-1} \left[\frac{6371}{\text{semimajor\_axis} \cdot (1 + \text{eccentricity})} \right]$$

southern_apogee_visibility =

$$\text{apogee\_latitude} - \sin^{-1} \left[\frac{6371}{\text{semimajor\_axis} \cdot (1 + \text{eccentricity})} \right]$$

When northernmost visibility latitude exceeds 90 degrees, apogee is visible at some latitudes while looking over the pole. While this is an excellent feature for an orbit to have, we'll limit the description of the northernmost apogee visibility to +90 degrees latitude for clarity. Likewise we limit the southernmost apogee latitude to -90 degrees.

North and South

The orbit's coverage range in north and south latitude is usually determined by the satellite's altitude at its northernmost and southernmost points, called the *antinodes*. The satellite's altitude at the northern and southern antinodes are computed as follows:

$$\text{north\_antinode\_altitude} = \frac{\text{semimajor\_axis} \cdot (1 - \text{eccentricity}^2)}{1 + \text{eccentricity} \cdot \sin(\text{argument\_of\_perigee})} - 6371$$

$$\text{south\_antinode\_altitude} = \frac{\text{semimajor\_axis} \cdot (1 - \text{eccentricity}^2)}{1 - \text{eccentricity} \cdot \sin(\text{argument\_of\_perigee})} - 6371$$

Using these altitudes, we then re-compute the coverage latitudes using the same procedure from Part 1 of this article:

$$\text{north\_latitude\_coverage} = \text{inclination} + \cos^{-1} \left[\frac{6371}{\text{north\_antinode\_altitude} + 6371} \right]$$

$$\text{south\_latitude\_coverage} = -\text{inclination} - \cos^{-1} \left[\frac{6371}{\text{south\_antinode\_altitude} + 6371} \right]$$

Once again, we limit the north and south coverage latitudes to +90 and -90 degrees, respectively.

While this appears to work for all practical examples, I cannot guarantee that it will work for 100% of the cases. If you find a case where apogee visibility is better than the corresponding north or south latitude coverage, use it instead.

Just to Summarize

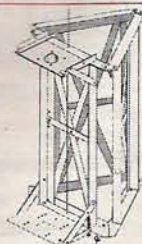
This article extended the orbital characteristics calculations from Part 1, giving a better idea of the north and south coverage range for elliptical orbits. You should be able to do the math with a scientific calculator. However, I have also updated the Excel spreadsheet on my Web page with the new calculations for those of you who have a computer and Internet access. In addition, I've done sample calculations in a "Worksheet for Analyzing Keplerian Elements" that appears on the following pages.

Happy number-crunching, and I hope this gives you a greater appreciation for the work your computer tracking program is doing!

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